ROUND I: Arithmetic - order of operations and evaluation of algebraic expressions

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Simplify: $-4\left(5+12\bullet\frac{1}{2}\right)\div 2+7+72\div 36$.

2. If x#y = x-xy and x##y = xy+y, evaluate [(2#3) # (6##1)] ## 2.

3. Evaluate to an integer or reduced fraction $1-\cfrac{1}{2-\cfrac{1}{3-\cfrac{2x-1}{2x+1}}}$ for $x=1$.

ANSWERS

1. (1 pt) _______________

2. (2 pts) _______________

3. (3 pts) _______________

Assabet Valley, Bartlett, Leicester

October 10, 2001 WOCOMAL VARSITY MEET

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Multiply to one simplified polynomial: $(x+1)(x^3 - x^2 + x - 1)$.

2. Find the ordered pair which satisfies $y = -2x + 5$ but does not satisfy $\frac{y+1}{x-3} = -2$

3. A math teacher / farmer bought some pigs for \$180. If each pig had cost a dollar more, he would have obtained 2 fewer pigs for the same money. How many pigs did he buy?

ANSWERS

1. (1 pt) ______________________________

2. (2 pts) (,)

3. (3 pts) ______________

Notre Dame, St. John's, Westboro

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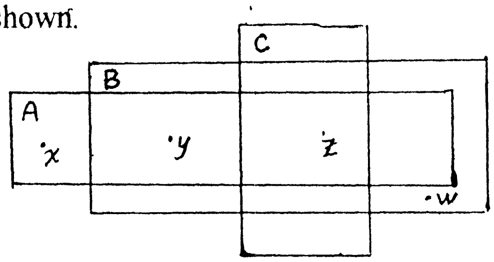
ROUND III : Set theory $\bar{A}$ denotes the complement of set A

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Sets A, B, and C include points w, x, y, and z as shown. Which points are in $(A \cap B) \cap \bar{C}$?



2. J = the set of integers
   A = $\{x \in J : 1 \le x \le 7\}$
   B = $\{x \in J : x = 2n, 0 \le x \le 5, n \in J\}$
   Specify $A \cap B$ by a list.

3. Al, Barney, and Chris each took the same100 item true-false test. Al had 70% of the items answered correctly. Barney had 78%, and Chris had 82% correctly answered. All of the items were answered correctly by at least one of the three. Al and Barney togerther answered 92% of the items correctly. Barney and Chris together answered 94% correctly. Al and Chris together answered 90% correctly. How many items were common correct answers for all 3?

ANSWERS

1. (1 pt) ____________________

2. (2 pts) { }

3. (3 pts) ____________________

Doherty, Quaboag, Worcester Academy

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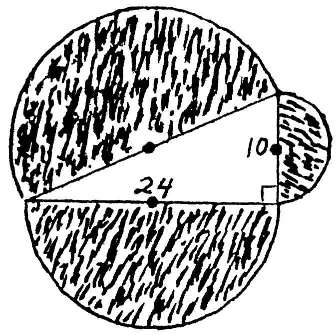
ROUND IV: Measurement - perimeter, area, volume

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The sides of a triangle have lengths in the ratio of 3:5:7 and the triangle has a perimeter of 90 cm. What is the length of the longest side? Include units.

2. A cubic foot of a certain material weighs 4 pounds. How much will 216 cubic inches of this material weigh, in pounds?

3. Find the area of the shaded region in terms of π. Do not approximate π. The dots are circle centers.



ANSWERS

1. (1 pt) ________________

2. (2 pts) ________________

3. (3 pts) ______________________________

Millbury, Shrewsbury, Tantasqua

ROUND V: Polynomial equations

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM. If complex numbers occur in your answers, express them in the form $a+bi$ (but if b = 0, omit the bi term).

1. For what value of k will the equation $5x^2+8x+k=0$ have a double root?

2. Solve: $(x^2+1)^2+2(x^2+1)=3$.

3. Solve: $2ix^2-2x-5i=0$.

ANSWERS

1. (1 pt) ______________

2. (2 pts) __________________________

3. (3 pts) __________________________

Algonquin, Leicester, Notre Dame

TEAM ROUND: Topics of previous rounds and open

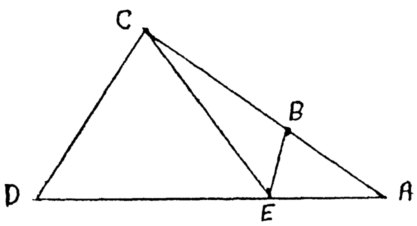
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET 2 points each

1. Evaluate: $\left(\frac{2}{3}+\frac{3}{4}+\frac{1}{2}\right)\div\left(\frac{7}{8}-\frac{5}{6}\right)$.

2. If $2@ = 3\&$ and $5@ + \& = 34$, find the value of $7@ - 2\&$.

3. Given: U = {whole numbers 20 through 40, inclusive}
A = {odd numbers}, B = {even numbers}, C = {multiples of 3},
D = {multiples of 5} and sets A, B, C, and D are all subsets of U.
Specify $\left[(C\cap D)\cup\left(\overline{A\cup B}\right)\right]\cup\left(\overline{A}\cap D\right)$ by listing its elements.

4. If $\frac{AB}{BC}=\frac{3}{5}$ and $\frac{AE}{ED}=\frac{4}{9}$, find the ratio $\frac{area\Delta ABE}{area\Delta ACD}$



5 If the arithmetic mean of two numbers is 6 and their geometric mean is 10 and an equation with the unspecified two numbers as roots is $x^2 + Bx + C = 0$, find B and C.

6. In a set of three races, a runner earns 5 points for a win, 3 points for second place, and 1 point for third; no ties allowed. At least how many points must one earn in the three races to be sure of earning more points than any other runner?

7. Find all values of x which satisfy $\frac{2}{x}+\frac{x}{2}=\frac{3}{x}+\frac{x}{3}$.

8. Adam takes 2 hours to do a job. Bob takes 3 hours. They worked together for a time and then Bob finished the job by working the same number of hours alone as he worked with Adam. How many hours did Bob work alone?

9. Solve for (p + q) in terms of r and s. Assume $r + s \neq 0$.
$rp + sq = r^2 + s^2$ and $sp + rq = 2rs$

ROUND I arith

1. 1 pt -13
2. 2 pts 50
3. 3 pts $\frac{5}{13}$

ROUND II alg 1

1. 1 pt x^4-1
2. 2 pts $(3,-1)$
3. 3 pts 20

ROUND III sets

1. 1 pt Y
2. 2 pts $\{2,4\}$ either order OK
3. 3 pts 54

ROUND IV meas

1. 1 pt 42 cm need units
2. 2 pts $\frac{1}{2}$ or .5 pound ← may omit
3. 3 pts $\cancel{120+}\ \frac{169}{\cancel{2}}\pi$ ~~not 383.165~~

   or ~~120 + 84.5π~~

ROUND V poly eq

1. 1 pt $3\frac{1}{5}$ or $\frac{16}{5}$ or 3.2
2. 2 pts $x = 0, 2i, -2i$ ± OK

   may omit
3. 3 pts $x = \frac{3}{2}-\frac{1}{2}i, -\frac{3}{2}-\frac{1}{2}i$

   1.5, .5i, $\frac{i}{2}$ etc. OK

TEAM ROUND 2 pts each

1. 52
2. 34
3. $\{20, 30, 40\}$ any order OK
4. $\frac{3}{26}$ or $3:26$
5. $B = -12$ $C = 100$
6. 13
7. $\sqrt{6}, -\sqrt{6}$ $\pm\sqrt{6}$ OK
8. $\frac{6}{7}$ hour $0.\overline{857142}$ OK

   may omit
9. $r + s$

October 10, 2001 WOCOMAL Varsity Meet BRIEF SOLUTIONS

ROUND I

1. $-4(5+6)\div 2+7+2$

$= \frac{-4(11)}{2}+9 = -2(11)+9 = -22+9 = -13$

2. $[(-4)\#7]\#\#2 = 24\#\#2 = 48+2 = 50$

3. $1-\cfrac{1}{2-\cfrac{1}{3-\frac{1}{3}}} = 1-\cfrac{1}{2-\cfrac{1}{\frac{8}{3}}} = 1-\cfrac{1}{2-\frac{3}{8}}$

$= 1-\cfrac{1}{\frac{13}{8}} = 1-\frac{8}{13} = \frac{5}{13}$

ROUND II

1. $x^4-x^3+x^2-x+x^3-x^2+x-1 = x^4-1$

2. For $x\neq 3$, the second equation becomes $y+1=-2x+6$, or $y=-2x+5$, the first equation. In this, when $x=3$, $y=-1$. Thus $(3,-1)$ fits the first equation, but not the second because of $\frac{0}{0}$.

3. Try integers with product 180 or:

$p = \#\text{pigs}$, c = cost each, $pc=180$ and

$(p-2)(c+1)=180$

$\therefore\ pc = pc-2c+p-2$ or $2c=p-2$ and $c=\frac{p-2}{2}$

$pc = 180 = \frac{p^2-2p}{2} \Rightarrow 0 = p^2-2p-360$

$(p-20)(p+18)=0 \Rightarrow p=20$

ROUND III

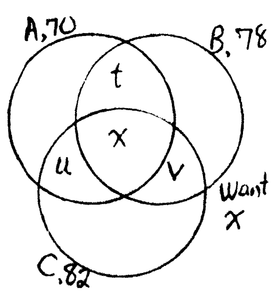
1. $A\cap B$ has pts y and z $\Big\}$ Their intersection has only y

$\bar{C}$ has pts x, y, and w

2. $A=\{1,2,3,4,5,6,7\}$

$B=\{0,2,4\}$ even

$\Big\}\ A\cap B=\{2,4\}$

3. 

Want x

$70+78-(t+x)=92 \Rightarrow t+x=56$

$78+82-(v+x)=94 \Rightarrow v+x=66$

$70+82-(u+x)=90 \Rightarrow \underline{u+x=62}$

$t+u+v+3x=184$

$70+78+82-(t+u+v+2x)=100$

$t+u+v+2x=130$

$x=54$

ROUND IV

1. $3x+5x+7x=90$

$15x=90$ and $x=6$

Longest side $=7x=42$ cm

2. $1\text{ ft}^3 = (12\text{ in})^3 = 1728\text{ in}^3$

Then $\frac{x}{4\text{ pounds}} = \frac{216}{1728}\ \left(=\frac{1}{8}\right)$

$x=\frac{1}{2}$ pound

3. By Pythag thm, hypotenuse $=26$

Radii $=5, 12, 13$

Shaded area $=\frac{1}{2}(25\pi+144\pi+169\pi)=169\pi$

ROUND V

1. Make $b^2-4ac=0$.

$64-20k=0 \Rightarrow k=\frac{64}{20}=\frac{16}{5}$

2. $x^4+2x^2+1+2x^2+2-3=0$

$x^4+4x^2=0$

$x^2(x^2+4)=0 \Rightarrow x=0, x=\pm 2i$

3. Multiply both sides of the given equation by $-i$ to get

$2x^2+2ix-5=0$.

Then $x=\frac{-2i\pm\sqrt{-4+40}}{4} = \frac{-2i\pm 6}{4}$

Simplified,

$x=\frac{3-i}{2}$ or $\frac{-3-i}{2}$

TEAM ROUND

1. $\left(\frac{2}{3}+\frac{3}{\cancel{4}_{2}}\cdot\frac{\cancel{2}}{1}\right)\div\left(\frac{21-20}{24}\right)=\left(\frac{4+9}{\cancel{6}}\right)\left(\frac{\cancel{24}^{\,4}}{1}\right)=$ [illegible]

2. Rearrange $2@-3\&=0$

$\underline{5@+\&=34}$

and add $7@-2\&=34$

Alt: Solve system; $@=6$ and $\&=4$

TEAM ROUND cont

3 $C \cap D = \{30\}$ mult of 3 and 5

$A \cup E = U$, so $\overline{A \cup B} = \phi$

$\bar{A} \cap D = \{20, 30, 40\}$ [illegible] mult of 5

$[\{30\} \cup \phi] \cup \{20, 30, 40\} = \{20, 30, 40\}$

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4 Ratio of areas of Δs with same heights = ratio of bases. Let $\alpha\Delta ABE$ = area of ΔABE

$$\frac{\alpha\Delta ABE}{\alpha\Delta ACD} = \frac{\alpha\Delta ABE}{\alpha\Delta ACE} \cdot \frac{\alpha\Delta ACE}{\alpha\Delta ACD} = \frac{AB}{AC} \cdot \frac{AE}{AD} = \frac{3}{8} \cdot \frac{4}{13} = \frac{3}{26}$$

---

5 Call the numbers u and v

Then $\frac{u+v}{2} = 6 \Rightarrow u+v = 12$

and $\frac{u}{10} = \frac{10}{v} \Rightarrow uv = 100$

Equation $(x-u)(x-v) = 0$

$$x^2 - (u+v)x + uv = 0$$
$$\left.\begin{aligned} x^2 - 12x + 100 &= 0 \\ x^2 + Bx + C &= 0 \end{aligned}\right\} \quad \begin{aligned} B &= -12 \\ C &= 100 \end{aligned}$$

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6 With points of $5+5+3=13$, the most that anyone else can get is $3+3+5=11$

With pts of $3+3+5=11$, someone else could also get 11 pts from $5+5+1$ An[illegible]

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7 $\frac{2}{y}$ [illegible] x [illegible] $\frac{2}{3}$

$\frac{x}{2}$ $\frac{x}{3}$ $\frac{3}{x} - \frac{2}{x}$

$\frac{x}{6} = \frac{1}{x} \Rightarrow x^2 = 6$ [illegible] $x = \pm\sqrt{6}$

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8. Let t = time together, hours

time for Bob to finish

$$\underbrace{\frac{t}{2} + \frac{t}{3}}_{\text{together}} + \underbrace{\frac{t}{3}}_{\text{Bob alone}} = 1$$

$3t + 4t = 6$

$t = \frac{6}{7}$ hour

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9 Add the given equations

$\underline{rp}$ [illegible] $+ \underline{rq} = r^2 + 2rs + s^2$

[illegible] $(r+s)^2$

$(r+s)(p+q) = (r+s)^2$

$p+q = r+s$